Diameter distribution models

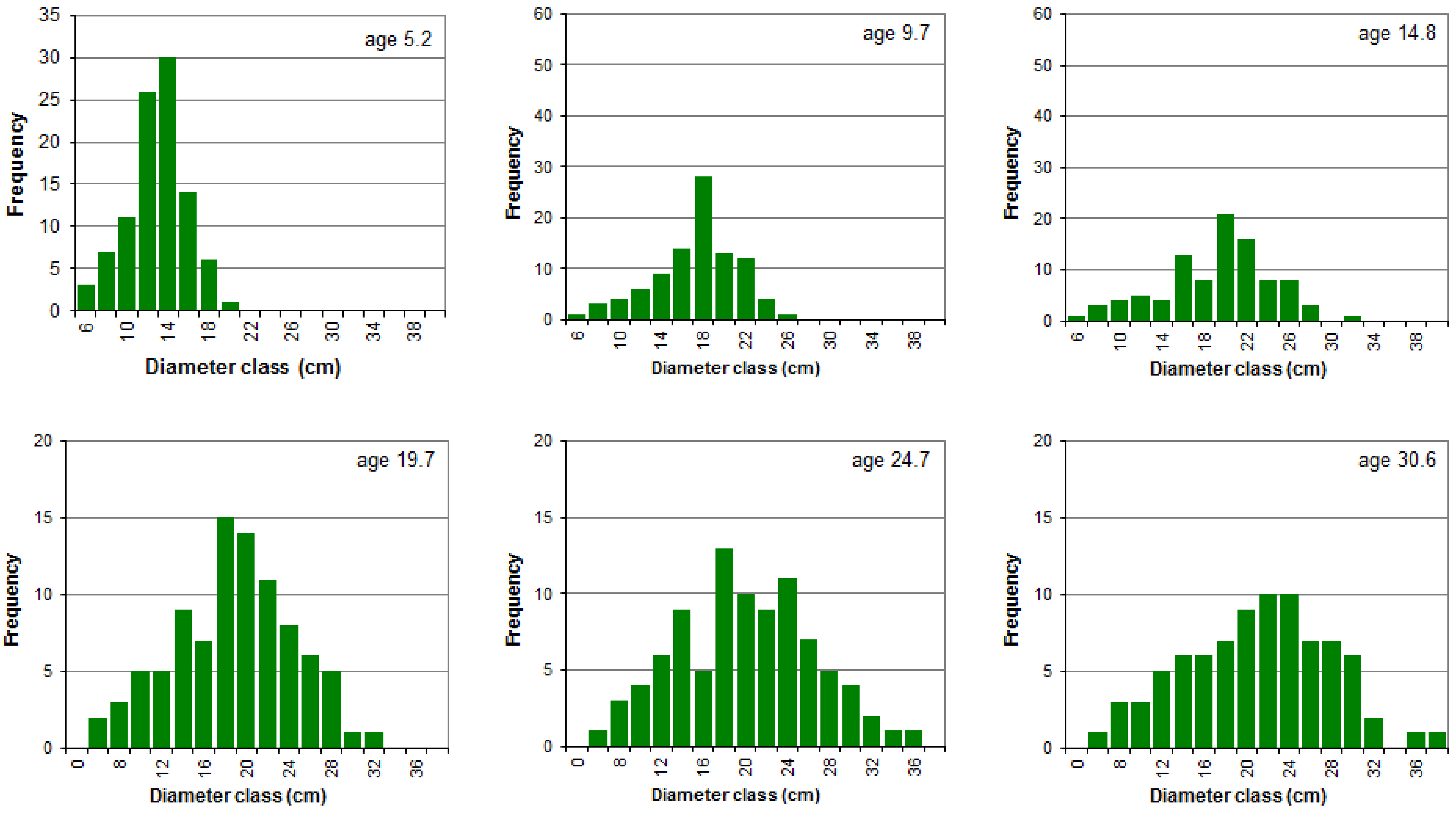
Diameter distribution models

- The idea behind diameter distribution models is:
  - To start by simulating the growth of some variables (principal variables):
    - dominant height
    - number of trees per ha
    - stand basal area estimation
    - some variables characterizing the diameter distribution such as the minimum diameter, some percentile of the diameter distribution or the variance of diameters (depending on the pdf used for diameter distribution)

Diameter distribution models

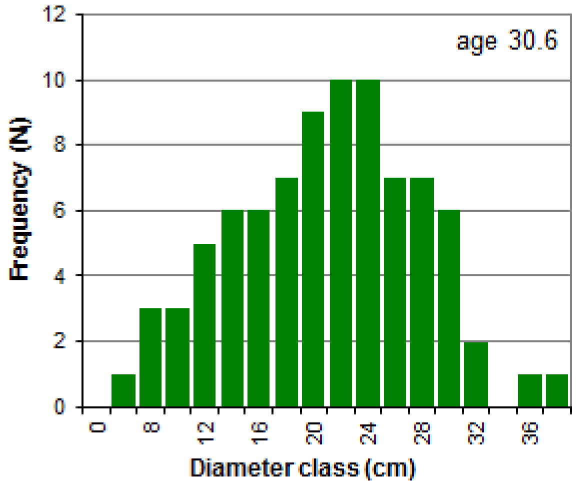
- The idea behind diameter distribution models is:
  - to estimate the distribution of trees by diameter classes (diameter distribution)
  - Usually the simulation of diameter distribution implies the need to predict other variables, namely minimum diameter and some percentile in the upper part of the distribution
  - to estimate stand volume (total and merchantable) from the diameter distribution, by using tree volume equations

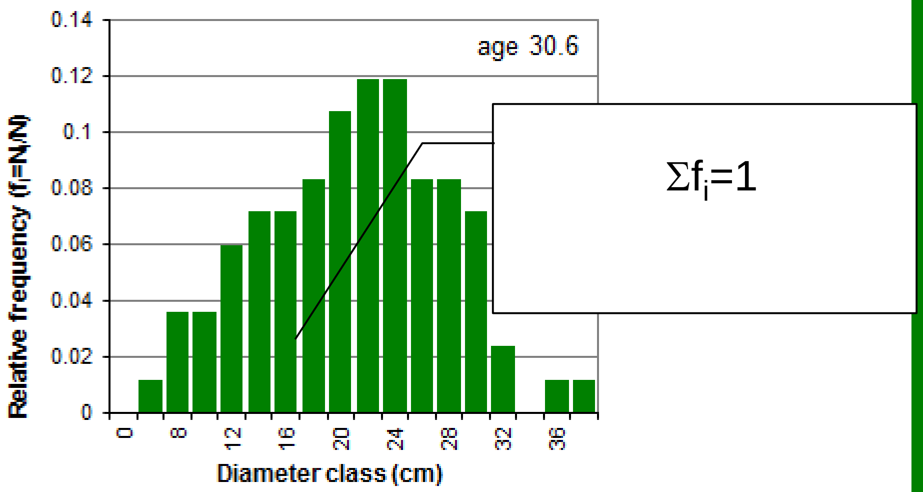
Diameter distributions of a permanent plot over time



Diameter distribution in relative frequencies

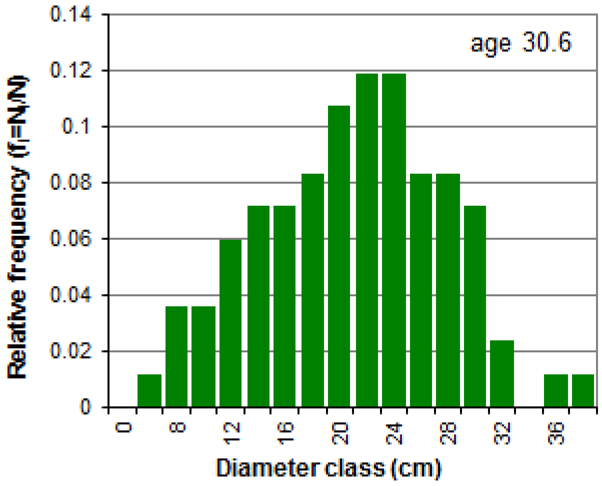
- The diameter distributions may be expressed in terms of relative frequencies by expressing the frequency each diameter class (N_i) relative to the total number of trees per ha (N)

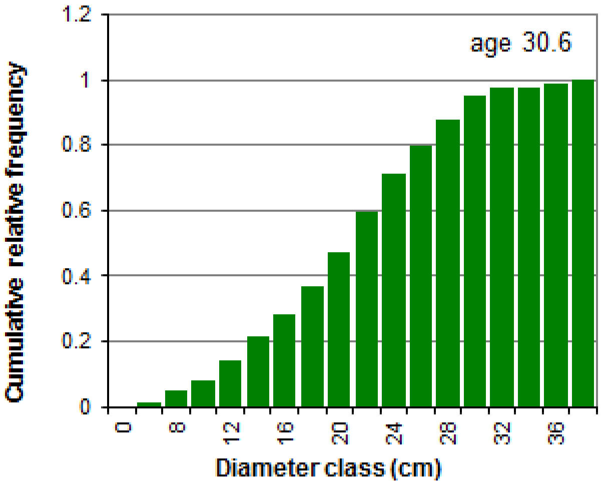




Diameter distribution in cumulative relative frequencies

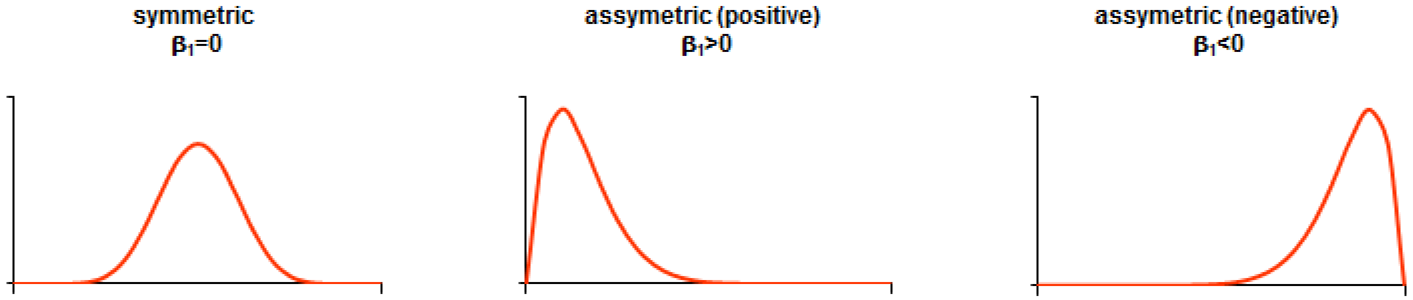
- The cumulative relative frequency of a diameter distribution leads to the empirical distribution function





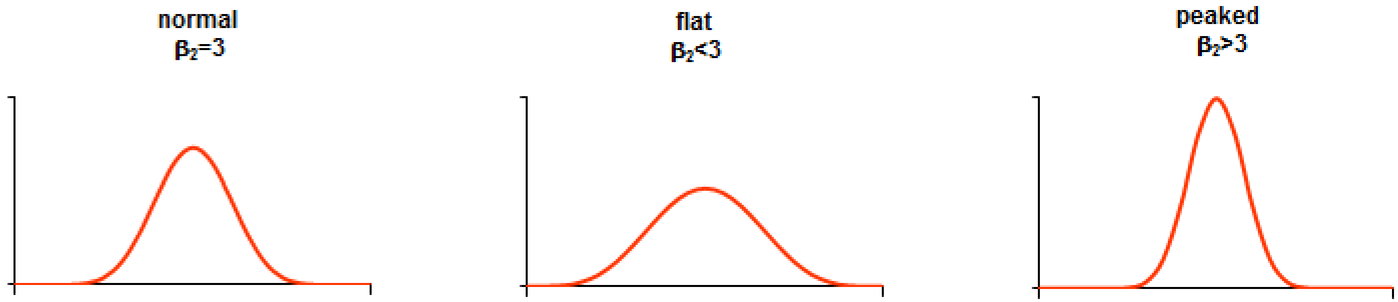
Modelling diameter distributions

- A typical diameter distribution for pure, even-aged stands is unimodal and slightly skewed
- Skewness coefficient (β_1) is used to measure the symmetry of a distribution



Modelling diameter distributions

- Diameter distributions for pure, even-aged stands can also be more or less flat
- Kurtosis coefficient (β_2) is used to measure flatness or peakdness of a distribution



Modelling diameter distributions

- Diameter distributions can be modelled by a variety of mathematical functions from the probability density functions (pdfs) type
- Probability density functions express the relative likelihood for a random variable to take on a given value
- The probability density function is nonnegative everywhere, and its integral over the entire space is equal to one
- The probability that the random variable takes a value<*x* is equal to the integral of the pdf from the start to *x*

Probability density functions (pdfs)

- A pdf is described by a mathematical expression that contains parameters
- The values of the parameters give a different shape to the pdf
- For instance, a normal distribution, the most well know pdf, has the following expression

$$f(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

that includes two parameters, the mean (μ) and the standard deviation (σ), and is designated by $N(\mu,\sigma)$

Probability density functions (pdfs)

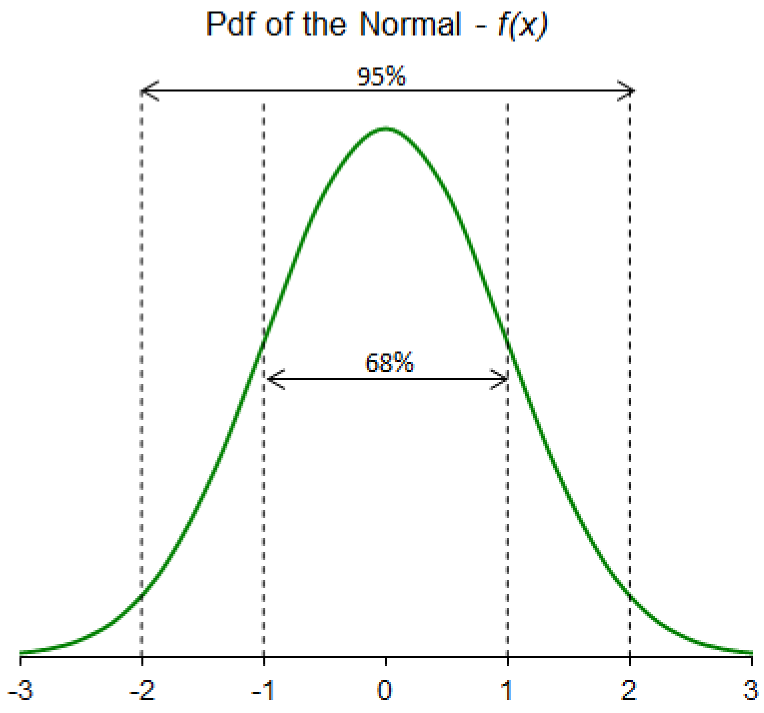
- Integrating the pdf produces the cumulative distribution function that, for the normal distribution is

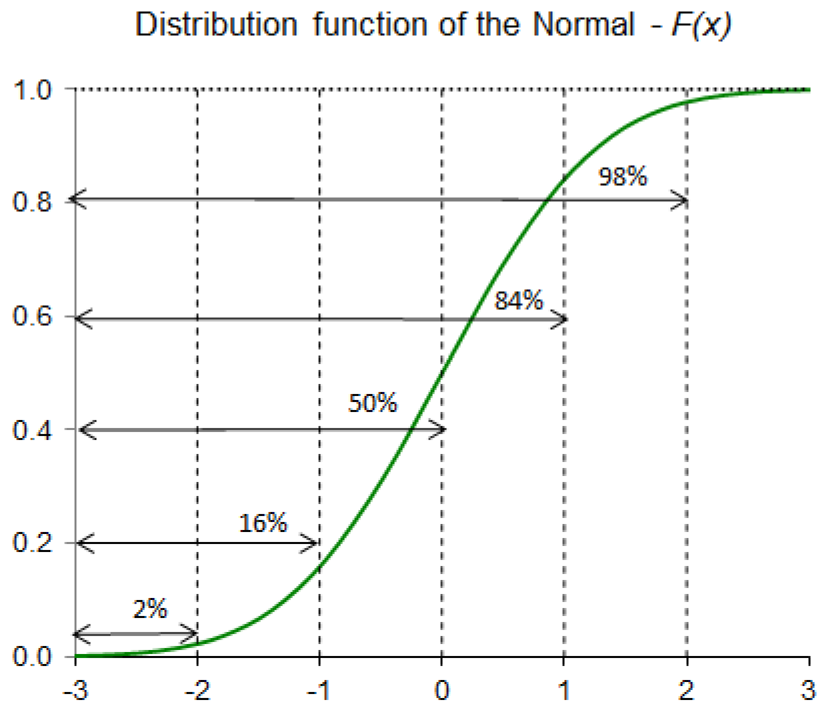
$$F(x;\mu,\sigma) = \text{Prob}(X <= x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

- The Normal distribution is not appropriate to model diameter distributions because of its symmetry

Probability density functions (pdfs)

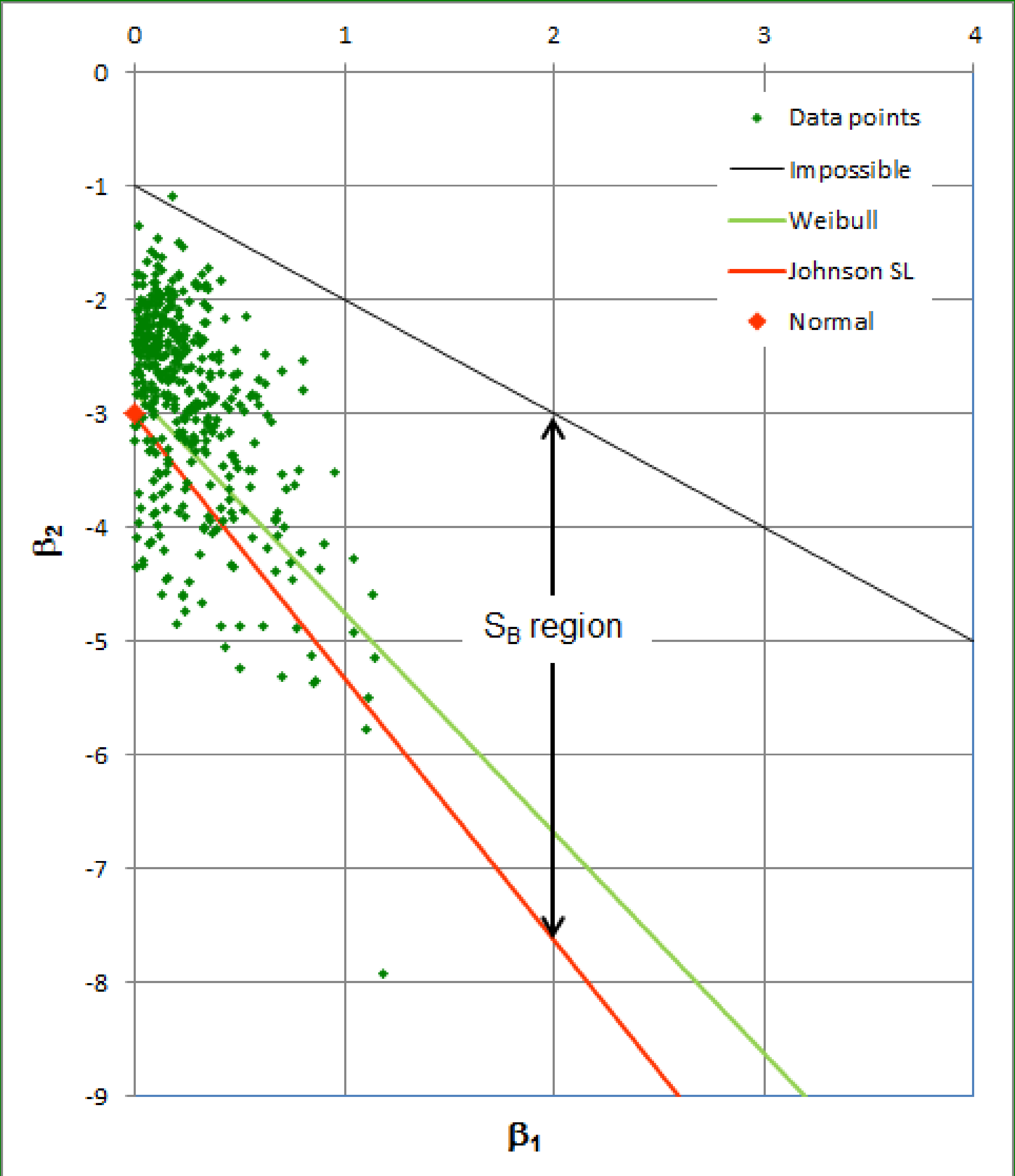
- The normal distribution is symmetric around the mean (μ)





Probability density functions (pdfs)

- Other pdfs, that can take different values for the pair (β_1, β_2), have been used for diameter distribution modelling



(β_1,β_2) values for the pdfs most used for diameter distribution modeling and for a set of eucalyptus permanent plots

Estimatos for β_1 and β_2:

$$\sqrt{b_1} = \sqrt{n}\,\frac{m_3}{m_2^{3/2}}$$

$$b_2 = n\,\frac{m_4}{m_2^2}$$

$$m_j = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^j}{n}, \qquad j = 2,3,4$$

n - number of observations

Weibull pdf

- The Weibull, one of the most used pdfs in diameter distribution modeling, is a three-parameter pdf

$$f(x) = \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] \quad (a <= x < \infty)$$

$$= 0 \quad \text{otherwise}$$

a – location parameter (related to the d_{min})

b – scale parameter (>0)

c – shape parameter (>0; if c>1 implies a inverse J shape; if c=3.6 is close to Normal; c<3.6 is right skewed; if c>3.6 is left skewed)

a+b is close to percentile 63% (P_{63}) of the distribution

Weibull pdf

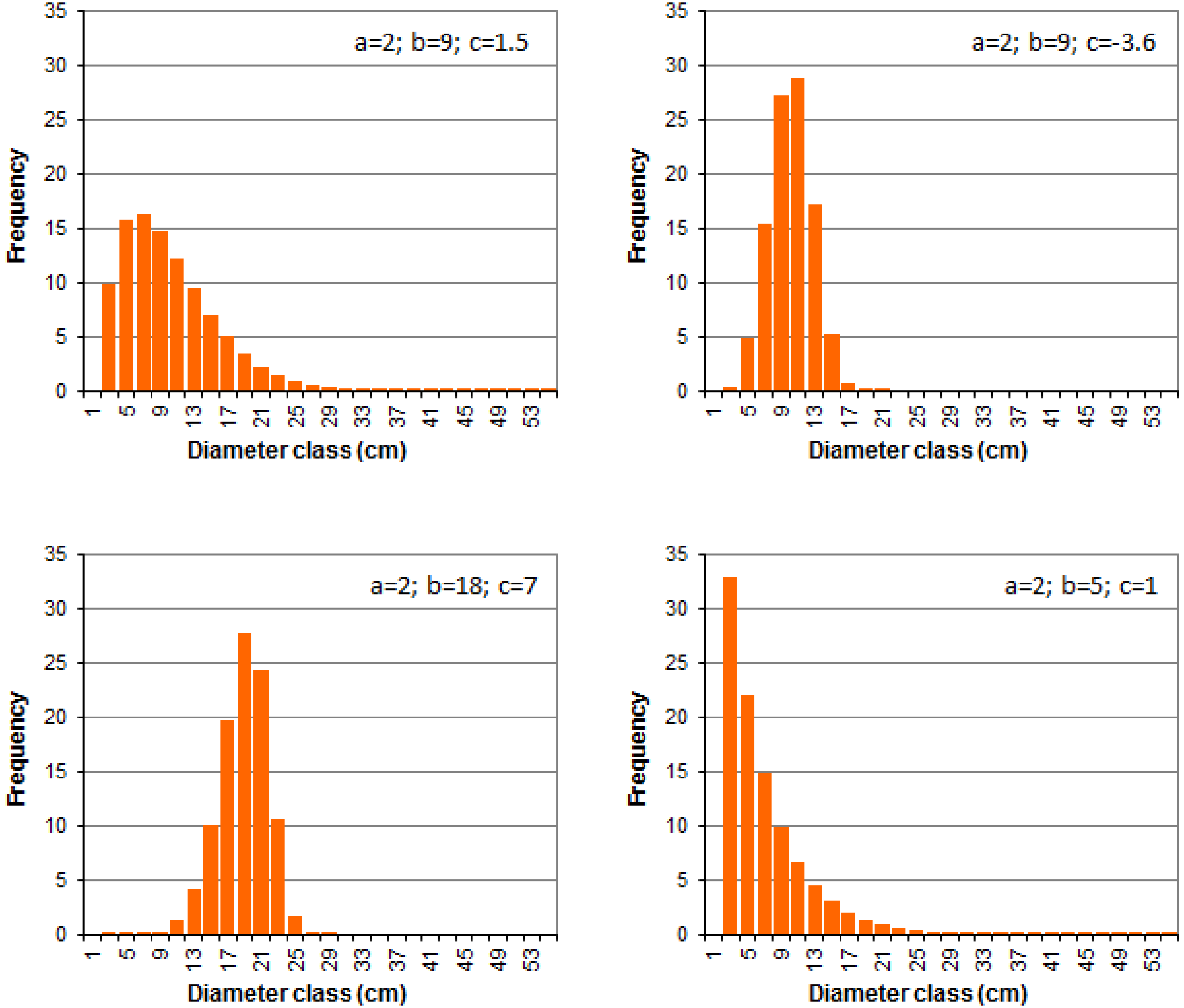
- Integrating the pdf produces the cumulative distribution function for the Weibull distribution

$$F(x) = 1 - exp\left[-\left(\frac{x-a}{b} \right)^{c} \right] \quad (a <= x < \infty)$$

$$= 0 \quad \text{otherwise}$$

- The Weibull distribution has the advantage of having a closed integral form which makes it very tractable

Weibull distribution - examples



The Johnson's SB system of pdfs

- The system of random variables generated by

$$Z = \gamma + \delta \; ln\left(\frac{X - \varepsilon}{\varepsilon + \lambda - X}\right)$$

$$\varepsilon < X < \varepsilon + \lambda$$

$$-\infty < \gamma < \infty$$

$$-\infty < \varepsilon < \infty$$

$$\lambda > 0$$

  is called the Johnson's SB system of distributions

- It is very flexible and can take several shapes

Johnson's SB system - examples