PBRAVO model – site index curves

- Dominant height growth/Site index curves

$$h_{dom} = S \; 10^{\left(0.380999 - 2.694076 \; t^{-1/2}\right)}$$

$$S = h_{dom} \; 10^{\left(-0.380999 + 2.694076 \; t^{-1/2}\right)}$$

PBRAVO model - initialization

- **Basal area**

$$G = 0.000984 \; h_{dom}^{2.103402} N^{0.797707} e^{-\frac{8.702327}{t}}$$

- **Percentile 93%**

$$P_{93} = 2.754927 \; h_{dom}^{1.445365} N^{-0.161445}$$

- **Minimum diameter**

$$d_{\min} = 12.302998 \; h_{dom}^{1.617991} N^{-0.687786}$$

PBRAVO model – prediction equations

- Height diameter curve

$$h = 1.891036 \; h_{dom}^{0.890695} G^{-0.146749} N^{0.075548} e^{\left(\frac{2.000723}{t} - \frac{11.96184}{d}\right)}$$

- Tree volume prediction
  - Total volume

$$v = 0.336 \; g h^2 e^{\left(\frac{0.94 d + 3.79 h}{d h}\right)}$$

  - Merchantable volume

$$\frac{di^2}{d^2} = 1.0080 e^{\left(\frac{0.94}{h} + \frac{3.79}{d}\right)} \left(1 - 2\frac{hi}{h} + \frac{hi^2}{h^2}\right)$$

PBRAVO model - projection

- Minimum diameter

$$d_{min_2} = 44.995291 \left(\frac{G_1^{t_1/t_2}}{N_1} \right)^{1/2} e^{0.062039(t_2 - t_1) + \left(1 - \frac{t_1}{t_2}\right)(-6.721703 + 0.039053\ h_{dom})}$$

- Percentile 93

$$P_{93_2} = 154.155144 \left(\frac{G_1^{t_1/t_2}}{N_1} \right)^{1/2} e^{-0.009172(t_2 - t_1) + \left(1 - \frac{t_1}{t_2}\right)(2.919335 - 0.034485\ h_{dom})}$$

PBRAVO model - projection

- Basal area growth

$$G_2 = G_1^{\frac{t_1}{t_2}} e^{\left(1-\frac{t_1}{t_2}\right)\left(4.178774+0.039053\ h_{dom}\right)}$$

PBRAVO model - projection

- Number of trees per ha (mortality)
  - Young unthinned stands (<15 years)

$$N = Npl^{\left(1-0.00400588\ t\right)}$$ or

$$N = 1324.485\ G^{0.23156} hdom^{-0.382656} e^{5.407995\ t^{-1}}$$

  - regularly thinned stands

$$N_2 = N_1$$

there is no mortality, just thinnings - the model cannot be applied to non-managed stands

PBRAVO model – simulation of thinnings

- **Residual minimum diameter**
  - **1st thinning** $d_{min_{res}} = 0.438034 + 1.047001\ d_{min}$
  - **Following thinnings** $d_{min_{res}} = 0.684456 + 1.07904\ d_{min}$

- **Residual percentile 93**
  - **1st thinning** $P_{93_{res}} = 0.121887 + 1.000760\ P_{93}$
  - **Following thinnings** $P_{93_{res}} = 1.22143 + 0.981559\ P_{93}$

PBRAVO model – simulation of thinnings

- Residual number of trees per ha
  - 1st thinning

$$N_{res} = N\left[1-\left(1-\frac{G_{res}}{G}\right)^{0.896268}\right]^{0.882856}$$

  - Following thinnings

$$N_{res} = N\left[1-\left(1-\frac{G_{res}}{G}\right)^{0.715054}\right]^{0.820574}$$

PBRAVO model – diameter distribution

- The simulation of the diameter distribution is based on the Weibull distribution

$$F(x) = 1 - e^{-\left(\frac{x-a}{b}\right)^c}$$

  - Parameters *a* e *b* are estimated with the following equations:

$$a = 0.5\ d_{min}$$

$$b = \frac{(x_p - a)}{\left[(-\ln(1-p))^{1/c}\right]}$$

PBRAVO model – diameter distribution

- After *a* and *b* being estimated, the *c* parameter is estimated by resolution, using numerical methods, of the following non-linear equation

$$a^2 + 2a\left(x_p - a\right)\frac{\Gamma\left(1+\frac{1}{c}\right)}{-\ln\left(1-p\right)^{1/c}} + \left(x_p - a\right)^2 \frac{\Gamma\left(1+\frac{2}{c}\right)}{-\ln\left(1-p\right)^{2/c}} - \mathrm{E}\left(x^2\right) = 0$$

PBRAVO model – stand volume prediction

- Stand volume is estimated from the diameter distribution using the estimated values of N and of the Weibull parameters a, b and c
  - Let's see an example for N=868, a=2.6, b=9.0 e c=3.2
  - Start by using the Weibull function to estimate the accumulated frequency of trees per dbh class

PBRAVO model – stand volume prediction

- **Estimate accumulated probabilities of trees to occur below each dbh class**

classe d	dcentral (cm)	dsup (cm)	Weibull P(d<=dsup)
[2.5, 7.5]	5	7.5	0.137
]7.5, 12.5]	10	12.5	0.751
]12.5, 17.5]	15	17.5	0.994
]17.5, 22.5]	20	22.5	1.000
]22.5, 27.5]	25	27.5	1.000
]27.5, 32.5]	30	32.5	1.000
]32.5, 37.5]	35	37.5	1.000

$$F(12.5) = 1 - e^{-\left(\frac{12.5-2.6}{9.0}\right)^{3.2}}$$

PBRAVO model – stand volume prediction

- **Number of trees per class and basal area**
  - **Multiplying the accumulated probabilities by N and making the differences between consecutive d classes one obtains the diameter distribution and the respective G**

dcentral (cm)	Weibull P(d<=dsup)	Nacum (ha^{-1})	N (ha^{-1})	G (m^2ha^{-1})
5	0.137	120	120	0.2
10	0.751	659	539	4.2
15	0.994	873	214	3.8
20	1.000	878	5	0.2
25	1.000	878	0	0.0
30	1.000	878	0	0.0
35	1.000	878	0	0.0
	Povoamento:		878	8.4

$$= \left(\frac{\pi}{4} \; (10/100)^2 \right) \; 539$$

PBRAVO model – stand volume prediction

- **Stand volume estimation**
  - **Predicting the height and volume of the average tree of each d class, it is possible to estimate volume per d class by multiplying tree volume by the number of trees per ha. Stand volume is estimated by summing up these values**

dcentral (cm)	N (ha^{-1})	h (m)	v (árvore)	V ($m^3 ha^{-1}$)
5.0	120	1.4	0.003848	0.5
10.0	539	4.6	0.021739	11.7
15.0	214	6.8	0.060050	12.8
20.0	5	8.4	0.119334	0.6
25.0	0	0.0	0.000000	0.0
30.0	0	0.0	0.000000	0.0
35.0	0	0.0	0.000000	0.0
	878			25.6

$= (0.021739)\ 539$

PBRAVO model – stand simulator

- The PBRAVO model is implemented in an interactive user-friend stand simulator